Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2019

Subject Code: 2130002 **Date: 30/05/2019**

Subject Name:Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Solve $(x+y-2)\,dx+(x-y+4)\,dy=0$	**03**
	(b)	Solve $(1+y^2)+(x-e^{-\tan^{-1}y})\dfrac{dx}{dy}=0$	**04**
	(c)	Expand $f(x)=\lvert\cos x\rvert$ as a Fourier series in the interval $-\pi<x<\pi$	**07**
Q.2	**(a)**	Define unit step function and unit impulse function. Also sketch the graphs.	**03**
	(b)	Solve $\left(\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y\right)=4\sin 2x$	**04**
	(c)	Find the series solution of $y''+xy'+y=0$ about the ordinary point $x=0$.	**07**
		OR	
	(c)	Find the Fourier series expansion for $f(x)$, if $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$, Also deduce that $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots\ldots=\dfrac{\pi^2}{8}$	**07**
Q.3	**(a)**	Using Fourier integral representation, show that $\displaystyle\int_0^\infty \frac{1-\cos\pi\omega}{\omega}\sin\omega x\,d\omega=\begin{cases}\dfrac{\pi}{2}, & 0<x<\pi\\ 0, & x>\pi\end{cases}$	**03**
	(b)	Solve $\left(\dfrac{d^2y}{dx^2}+y\right)=x^2\sin 2x$	**04**
	(c)	Solve by method of variation of parameters $\left(\dfrac{d^2y}{dx^2}+9y\right)=\dfrac{1}{1+\sin 3x}$	**07**
		OR	
Q.3	**(a)**	Find Laplace transform of $te^{at}\sin at$	**03**
	(b)	Solve $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=5e^x-\sin 2x$	**04**

(c) Solve

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$$ 07

Q.4 (a) Find the orthogonal trajectories of the curve $y=x^2+c$ 03

(b) Find the Laplace transform of (i) $\cos(at+b)$ 04

(ii) $\sin^2 3t$

(c) State convolution theorem and apply it to evaluate 07

$$L^{-1}\left(\frac{s^2}{\left(s^2+4\right)^2}\right)$$

OR

Q.4

(a) Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4y=0$ 03

(b) Find Half range cosine series for $f(x)=(x-1)^2$ in the interval $0<x<1$ 04

(c) Solve $y''+4y'+3y=e^{-t}, y(0)=y'(0)=1$ using Laplace transform. 07

Q.5 (a) Form the partial differential equation by eliminating the arbitrary constants from $z=ax+by+a^2+b^2$ 03

(b) Solve $(y-z)p+(x-y)q=z-x$ 04

(c) Solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$, where $u(x,0)=4e^{-x}$ using the method of separation of variables. 07

OR

Q.5 (a) Form the partial differential equation by eliminating the arbitrary function from $f(x^2+y^2, z-xy)=0$ 03

(b) Solve $\log\left(\frac{\partial^2 z}{\partial x\partial y}\right)=x+y.$ 04

(c) A bar with insulated sides is initially at temperature 0^0C, throughout. The end $x=0$ is kept at 0^0C and heat is suddenly applied at the end $x=l$ so that $\frac{\partial u}{\partial x}=A$ for $x=l$, where A is a constant. Find the temperature function. 07

****